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CARGO TRANSFER AT SEA - THE PENDULATION
OF LOADS SUSPENDED FROM SHIPBOARD
CRANES

Harry S. Zwibel, et al

Naval Civil Engineering Laboratory
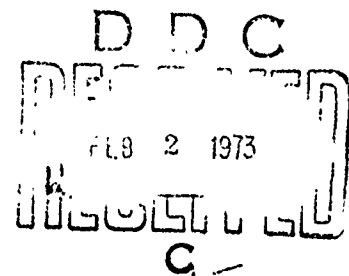Port Hueneme, California

December 1972

Technical Note N-1257

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NAVAL CIVIL ENGINEERING LABORATORY
Port Hueneme, California 93043

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YF 38.536.005.01.013

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A theory has been developed which could aid Navy materials handling specialists in their effort to evaluate load transfer systems for a modular port facility. The theory predicts the horizontal response of an unrestrained, wire suspended load in regular and random seas. The line length is allowed to vary with time, hence the resulting load response in random seas is characterized as a non-stationary random process.

The analysis is used to predict the motion of a load freely suspended from the boom of a Navy 100-ton floating crane. The results from the analysis and from full scale tests at sea confirm the fact that motion of unrestrained loads is a serious problem in even moderate sea states. Taglines or other means of restraint will be required from inception through completion of each load transfer.

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Cargo						
Materials Handling						
Loading Procedures						
Cranes (hoists)						
Suspending (hanging)						

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INTRODUCTION

The Naval Civil Engineering Laboratory is developing equipment and
techniques for the offshore discharge of containerships under the
sponsorship of the Naval Facilities Engineering Command (NAVFAC). This
effort is in support of the Navy's mobile advanced base port concept
termed Expeditionary Logistics Facility (ELF).

As part of this program, several analytical models have been
developed which are aiding materials handling, mooring and fender
specialists in their evaluation and selection of ELF hardware. One
model, code named RELMO, is used to compute the relative motions between
linearly moored, slender vessels in both regular and random seas*.
This analysis has been used, for example, to compute: (1) the relative
horizontal and vertical motion between two moored ocean-going vessels
in shoal water, (2) the relative horizontal and vertical motion between
a moored ship and an unmoored beach discharge lighter and (3) the shoal
water surge response of vessels as a function of mooring stiffness.

Data generated by the ship motion analysis (in the form of crane
boom tip displacement operators) can be used as input in a load pendu-
lation analysis** In this initial investigation of load pendulation,
an explicit formula was obtained for the significant amplitude of un-
restrained load motion when the floating crane platform was exposed to
random head seas. The analysis was limited, however, to treating the
pendulation of a load in one plane, and it was further assumed that the
line supporting the load was fixed in length.

An improved load pendulation analysis has now been developed and
is the subject of the present report. Since the effects of raising and
lowering the load are considered in the analysis, it more realistically
models load pendulation during cargo handling operations at sea. The
point of load suspension (boom tip) is assumed to move in a horizontal
plane due to crane platform surge and pitch for bow-on waves; sway and
roll for beam-on waves. However, only bow-on waves are considered in
this report.

The analyst has the choice of computing load motion with respect
to a moving frame of reference, the boom tip, or, as has been done for
most of the examples presented herein, the load motions can be computed
for a fixed reference in space.

*Naval Civil Engineering Laboratory. Technical Note N-1183: The Relative
Motion Between Ships in Random Head Seas, by D. A. Davis and H. S. Zwibel
Port Hueneme, California, Sep 1971.
**Naval Civil Engineering Laboratory. Technical Note N-1187: Motion
of Freely Suspended Loads Due to Horizontal Ship Motion in Random Head
Seas, by H. S. Zwibel. Port Hueneme, California, Oct 1971.

Results for cargo pendulation aboard a typical Navy yard crane are presented and discussed. The versatility of the analysis, as well as its limitations, will become apparent after reviewing these results.

THEORY

A heavy load suspended by a wire from a boom acts like a pendulum and a swinging motion of the load can be initiated by horizontal accelerations of the boom. The mathematical analysis of this forced pendulation is complicated for several reasons: first, the length of the pendulum is changing with time due to raising and lowering of the load; second, the boom acceleration is a random function of time. This forced pendulation problem, therefore, falls under the category of non-stationary stochastic processes. In fact, if one considers the possibility of large amplitude oscillations, then the equation of motion is also non-linear. There are analytical means for determining the statistics of stationary stochastic processes; unfortunately there are no comparable methods available for general non-stationary random processes. One must use the "brute force" method of simulating the random input function and numerically integrating the equation of motion to obtain the output. In order to get statistical information it is necessary to repeat this procedure for a number of inputs. Statistical accuracy increases with the total number of simulations.

The equation of motion is obtained by equating the time rate of change in the load angular momentum to the applied torque. This yields the following equation:

$$\frac{d^2\theta(t)}{dt^2} = 2\,\frac{dL(t)}{dt}\,\frac{1}{L(t)}\,\frac{d\theta(t)}{dt} - \frac{g}{L(t)}\left[\sin \theta(t) + \frac{1}{g}\,\frac{d^2X_s(t)}{dt^2}\,\cos \theta(t)\right] \tag{1}$$

where

    t = time

    $\theta(t)$ = angle of pendulum with respect to the vertical

    $L(t)$ = length of pendulum (this is a deterministic, specified function of time)

    $X_s(t)$ = horizontal position of boom (the attachment point for the pendulum)

    g = acceleration of gravity

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For a given $L(t)$ and $X_s(t)$ it is a simple matter to numerically integrate the above equation. The complication arises because of the random nature of $X_s(t)$.

This random boom motion can be numerically simulated with standard Monte Carlo techniques.[*] However, before the simulation can be conducted, certain assumptions of the process must be made concerning the boom motion, namely that the sea elevation is a stationary Gaussian process and that the ship response is a linear function of the ocean waves. The horizontal boom motion, $X_s(t)$, then becomes a stationary Gaussian random process with a calculable power spectral density function.

The method used is the most "intuitive" of the various possible approaches. The desired stochastic variable, e. g., the sea surface elevation is represented as a sum of waves each with a different frequency. The phase of each wave is independently chosen at random from the uniform distribution, and the amplitude of each wave is chosen so that the power spectral density function for the process is a specified function. Mathematically, then, the simulation of $X_s(t)$ is described by the following equation:

$$X_s(t) = 2 \sum_{n=1}^{N} \left[S(\tilde{\omega}_n) \Delta_n \right]^{\frac{1}{2}} \cos \left(\tilde{\omega}_n t + \varphi_n \right) \tag{2}$$

where

$\tilde{\omega}_n$ = is the circular frequency at the midpoint of nth frequency interval.

Δ_n = is the width of the nth interval.

φ_n = is the randomly chosen phase of the nth wave.

$S(\tilde{\omega}_n)$ = is the power spectral density for the horizontal motion of the boom.

N = is the total number of individual waves.

The boom acceleration obtained from $X_s(t)$ is

$$\ddot{X}_s(t) = -2 \sum_{n=1}^{N} \tilde{\omega}_n^2 \left[S\left(\tilde{\omega}_n\right) \Delta_n \right]^{\frac{1}{2}} \cos\left(\tilde{\omega}_n t + \varphi_n\right) \tag{3}$$

and is used as the forcing acceleration for the load pendulation.

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[*]Columbia University. Department of Civil Engineering and Engineering Mechanics. Technical Report No. 12: Simulation of Multivariate and Multidimensional Process II, by M. Shinozuka and C. M. Jan, New York, April 1971.

It should be noted that a given set of N random phases yields a simulation of $X_s(t)$ that is a deterministic function of time. It represents one member from an ensemble of boom motions. The pendulation time history obtained by integrating the equation of motion is, therefore, only representative, and statistical inferences must be made by sampling many members from the ensemble.

RESULTS AND DISCUSSION

The theory is best demonstrated by applying it to a typical open sea cargo transfer problem. Consider, then, the case of a Navy 100-ton yard crane, e. g., the YD-225, unloading containers from a ship moored in the open sea. Both ship and crane are headed bow-on into the incident unidirectional sea. A lighter lies between the crane and ship and serves as a receiving platform for off-loaded containers. The crane lifts the upper most container from a stack of three resting on the deck of the ship, raises the container 10 feet at a constant line rate of 79 fpm and then immediately lowers the container at the same line rate to the well deck of the lighter. To actually accomplish this transfer, the crane boom must either be rotated or raised to position the container over the lighter. The theory, however, does not account for changes in boom position during a load transfer and this variable is disregarded in the analysis that follows.

Figure 1 depicts the crane, lighter and ship at the instant that the load is lifted. The crane boom is positioned normal to the barge longitudinal axis and the point of line suspension is 61 feet above the top of the container. For the sake of simplicity, the center of gravity of the container, spreader bar and hook is assumed to be located at the geometric center of the 8 foot high container; thus the effective line length at the beginning of the load cycle is 61 + 4 = 65 feet. The well deck of the lighter is one foot below the water surface, hence the effective line length when the container is released is 116 feet. The problem statement is completed by noting that the mean water depth at the unloading site is assumed to be 100 feet and that the crane barge moorings are assumed to have no effect on the barge motion for the frequency range of interest.

Motion in Regular Waves

Response Operators for Load Suspension Point. Figures 2 through 7 depict the load suspension point motion response amplitude operators. Two traces appear in each figure: the solid line represents the motion of the load suspension point with respect to a fixed frame of reference, while the boken line is the relative motion response amplitude operator between the crane boom suspension point and the well deck of a lighter, an aluminum LCM-8. These results were generated by the NCEL relative

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ship motion analytical model (code named RELMO) using as input the vessel characteristics appearing in Table 1.*

Table 1.   Characteristics of Vessels Used in
Motion Analysis

Vessel	Displacement (LT)	Length Overall (ft)	Beam (ft)	Mean Draft (ft)
YD-225	1540	140	70	6.0
LCM-8	47.3	71.5	21.2	2.4

Load Pendulation. The load displacement amplitude operators for five different wave periods are shown in Figure 8. The results were obtained from an analytical model (code named SWING) which computes load oscillation for regular period horizontal excitation of the load suspension point. The abscissa in each graph is expressed as time (in seconds) from the beginning of the load transfer cycle. Thus at time 7.6 seconds the load has been lifted 10 feet from the containership deck, and at 53.9 seconds from lift-off the load is resting on the well deck of the lighter. The ordinate of each trace is expressed as the ratio of load displacement amplitude over boom displacement amplitude.

For a boom period of 6 seconds (or less), the motion of the load is comparatively small, and there is no evidence of resonant behavior. Considerably more motion occurs for 8-second period excitation, and a maximum response occures for excitation of 9.5 seconds. In the latter case, the unrestrained load is seen to pendulate with an amplitude equal to 15 times the boom displacement amplitude. Resonant behavior is far less evident in the plot for 12-second excitation, and at the 16 seconds the load response is comparable in magnitude to that noted for 6 second excitation.

If one scans the frequency dependent load displacement operators in Figure 8 and selects from each of these the maximum swing amplitude that occurs during each load transfer cycle (regardless of when it occurs during the cycle), then a plot can be formed such as the lower trace in Figure 9. Two additional traces for slower line speeds, 50 fpm and 25 fpm, are also shown in this figure. It is clear from these results that faster line handling rates result in a lowering of the load displacement amplitude operator.

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*It is the horizontal displacement amplitude operator for the absolute motion load suspension point (Figure 5) which is used later of in computing the random motion of the pendulating load.
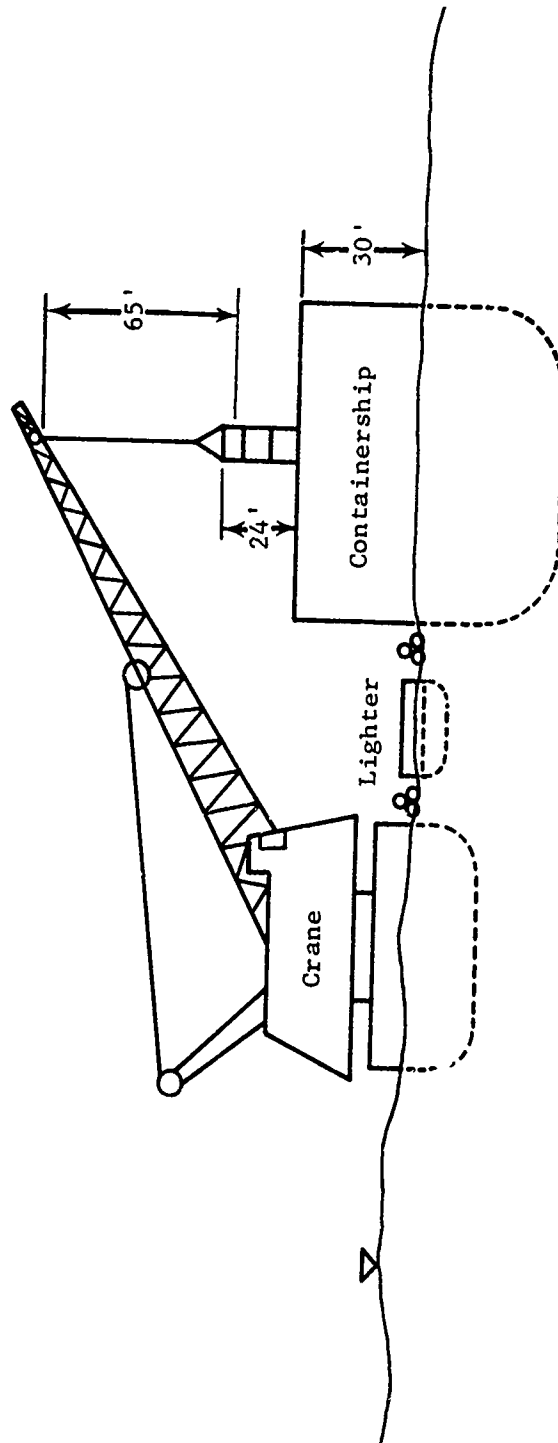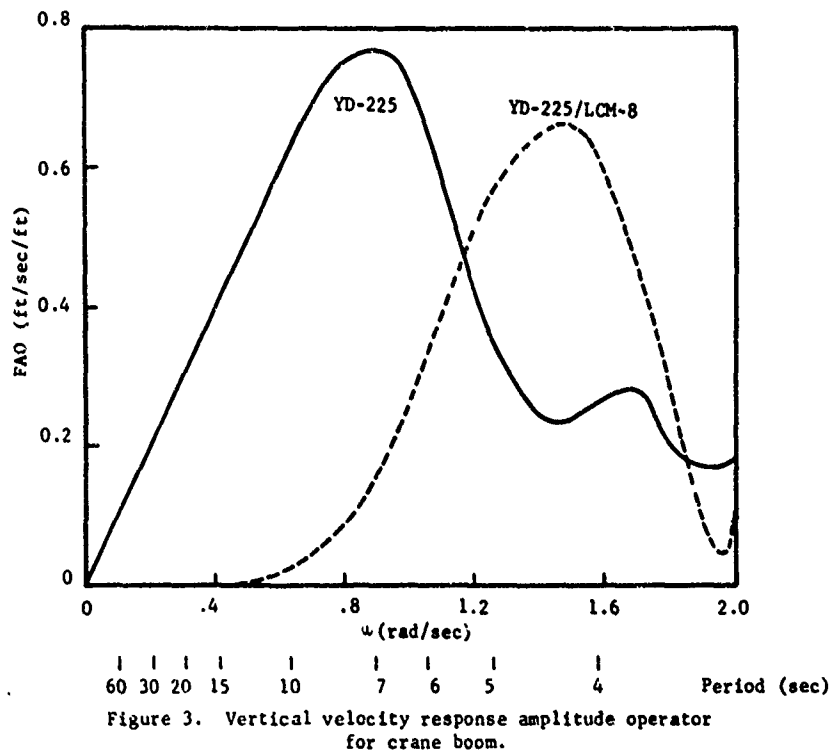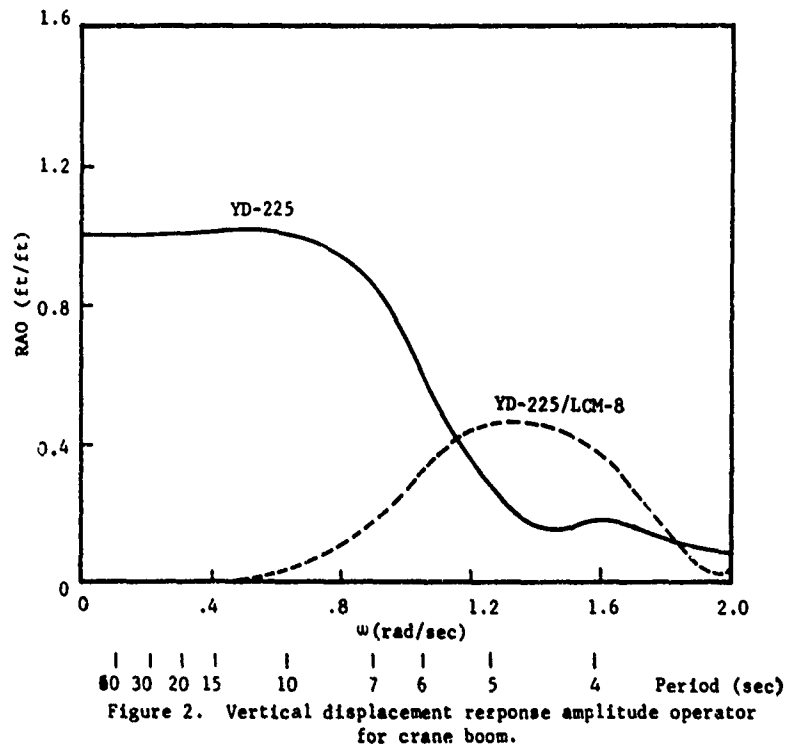
Figure 1. Unloading containernized cargo with 100-ton Navy yard crane.

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Figure 2. Vertical displacement response amplitude operator for crane boom.

Figure 3. Vertical velocity response amplitude operator for crane boom.
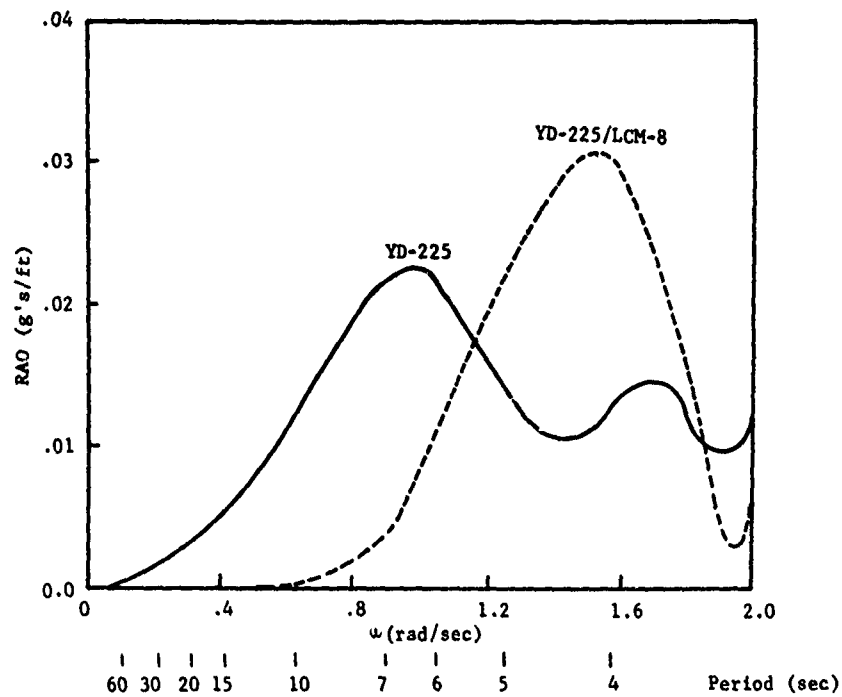
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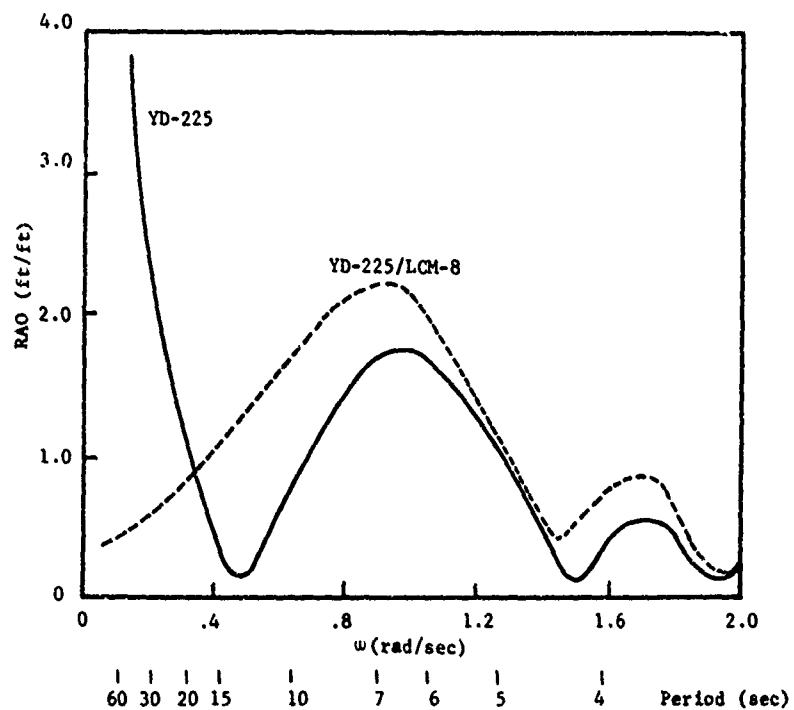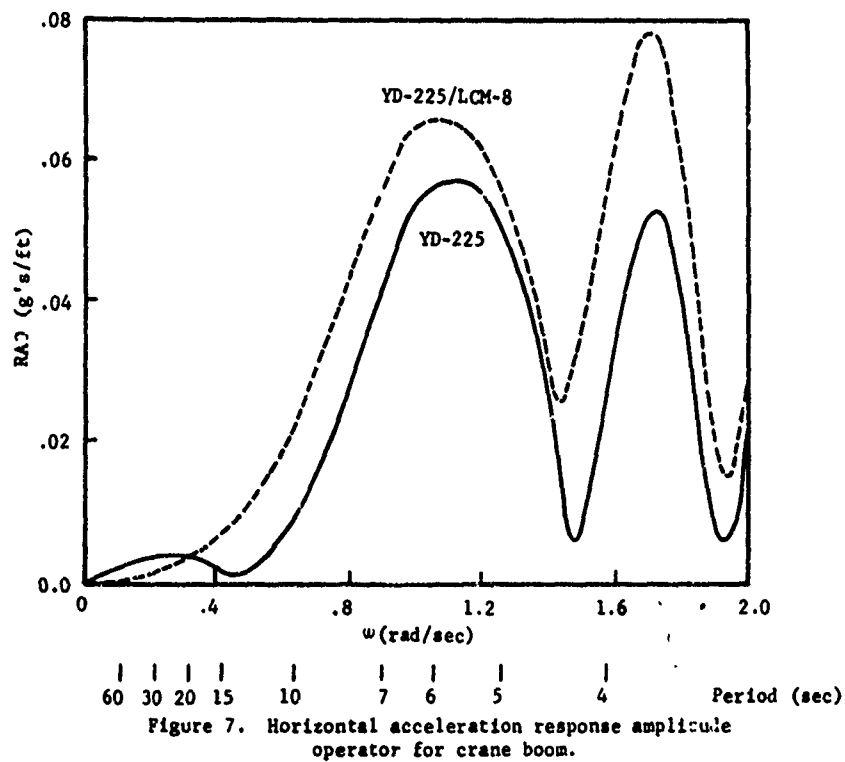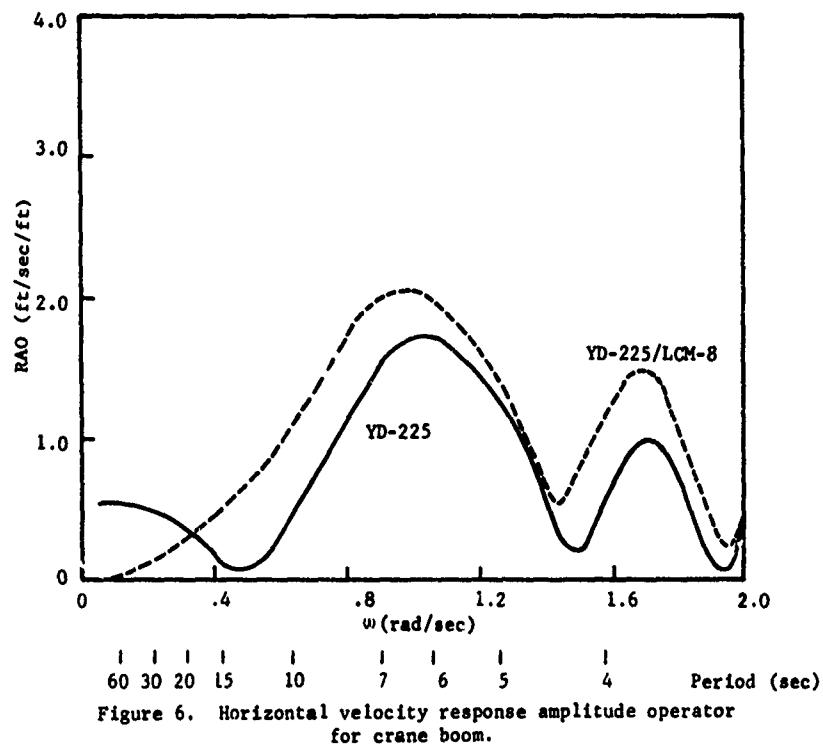Figure 4. Vertical acceleration response amplitude operator
for crane boom.

Figure 5. Horizontal displacement response amplitude operator
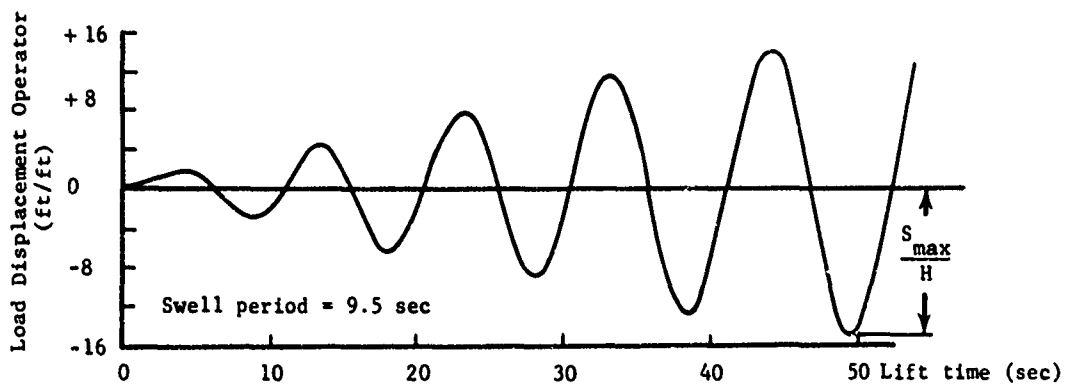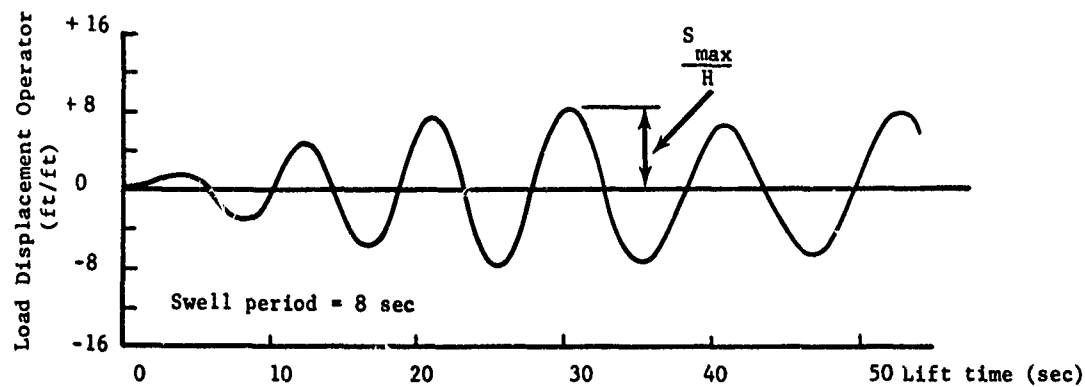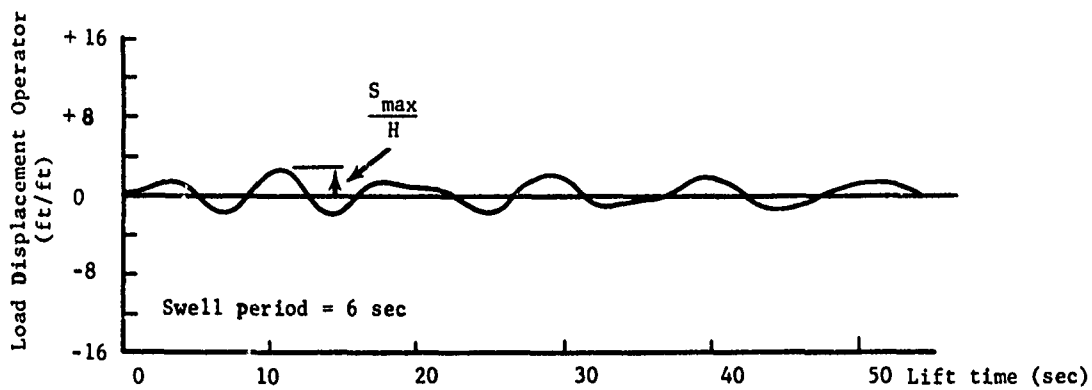for crane boom.

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Figure 6. Horizontal velocity response amplitude operator for crane boom.

Figure 7. Horizontal acceleration response amplitude operator for crane boom.

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Figure 8. Absolute load displacement in regular swell.

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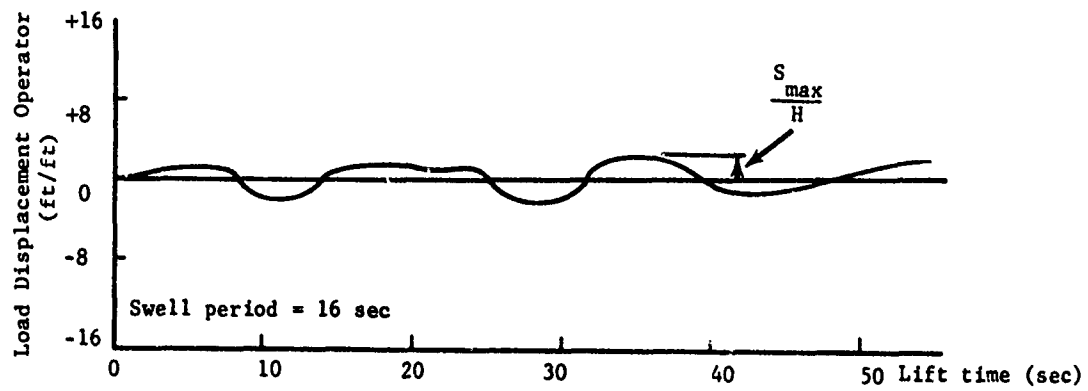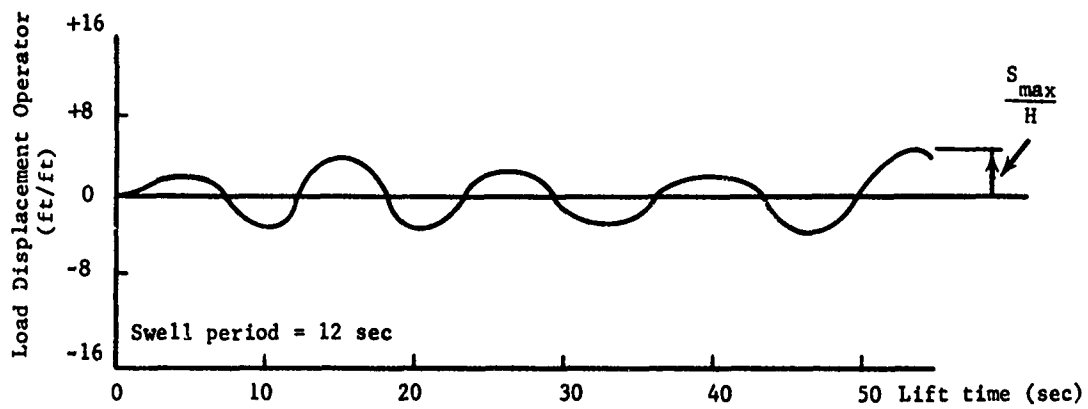
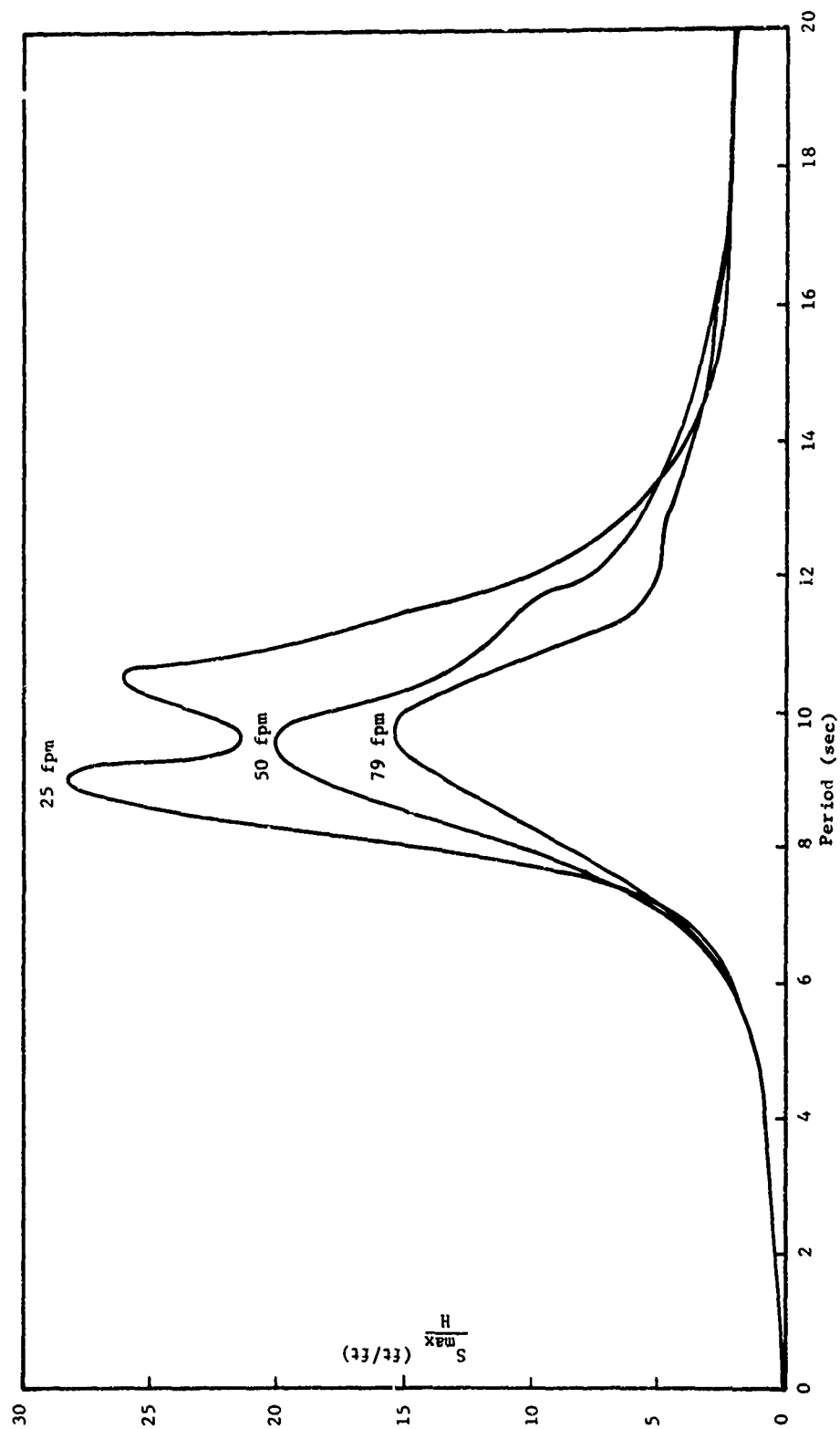Figure 8. Absolute load displacement in regular swell (Ccnt).

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Figure 9. Maximum load amplitude in regular swell.

The product of the boom tip horizontal displacement amplitude operator and the maximum load displacement amplitude operator produces the useful result shown in Figure 10. It is apprent from this plot that regular swell having a period of 9 seconds produces the maximum load amplitude, about 14.5 feet per foot of wave amplitude.

Motion in Random Waves

Response of Load Suspension Point. Estimates of the load suspension point motion in random seas were computed using the NCEL ship motion analysis.[*] The results appear in Figures 11 through 16 as plots of the average of the 1/3 highest displacement amplitude, i. e., significant amplitude, as a function of the deep water significant wave height, $H_{1/3}$. As before, for the estimates of load suspension point displacement amplitude operators, the results are plotted for both the absolute motion of the suspension point (solid line) and the relative motion between this point and the well deck of an LCM-8 lighter (broken line).

Load Response. As noted earlier, the solution of Equation (1) for the angular deflection of a wire suspended load in a random sea (with variations allowed in the suspension line length) was complicated by the dependency of this solution on the random nature of $X_s(t)$, the time dependent horizontal boom displacement. The function $X_s(t)$, however, can be simulated as shown in Equation (2), and the result in turn used to simulate the motion of the load, the independent variable of interest.

Thus, with the same load cycle as before (line speed = 79 ipm) and with random selection of wave phase angles for each simulation, a series of plots can be generated such as those which appear in Figures 17 and 18. These two simulations from the infinitely large ensemble of simulations differ markedly which illustrates the dependency of the solution on the random selection of wave phase angles. Obviously, accurate statistical estimates of the load motion are impossible without a substantial number of motion simulations. On the basis of nine simulations for each of three different line speeds ($L_{(t)}$ = 25.0, 50.0 and 79.0 fpm), the estimates for S_{max} (maximum load amplitude per load cycle) are plotted in Figure 19. It is apparent that the effect of increasing the crane line speed is to reduce the magnitude of load oscillation, although the reduction in magnitude is not as great as that noted earlier for pendulation in regular waves (Figure 9).

For different values of significant wave height, $H_{1/3}$, load motion simulations were made to determine the relationship between maximum load amplitude and sea state. The results, for a line speed of 79 fpm, appear in Figure 20. Although only five simulations were made for each value of $H_{1/3}$, it is apparent that S_{max} (and the spread in predicted amplitude as well) increases with sea state.

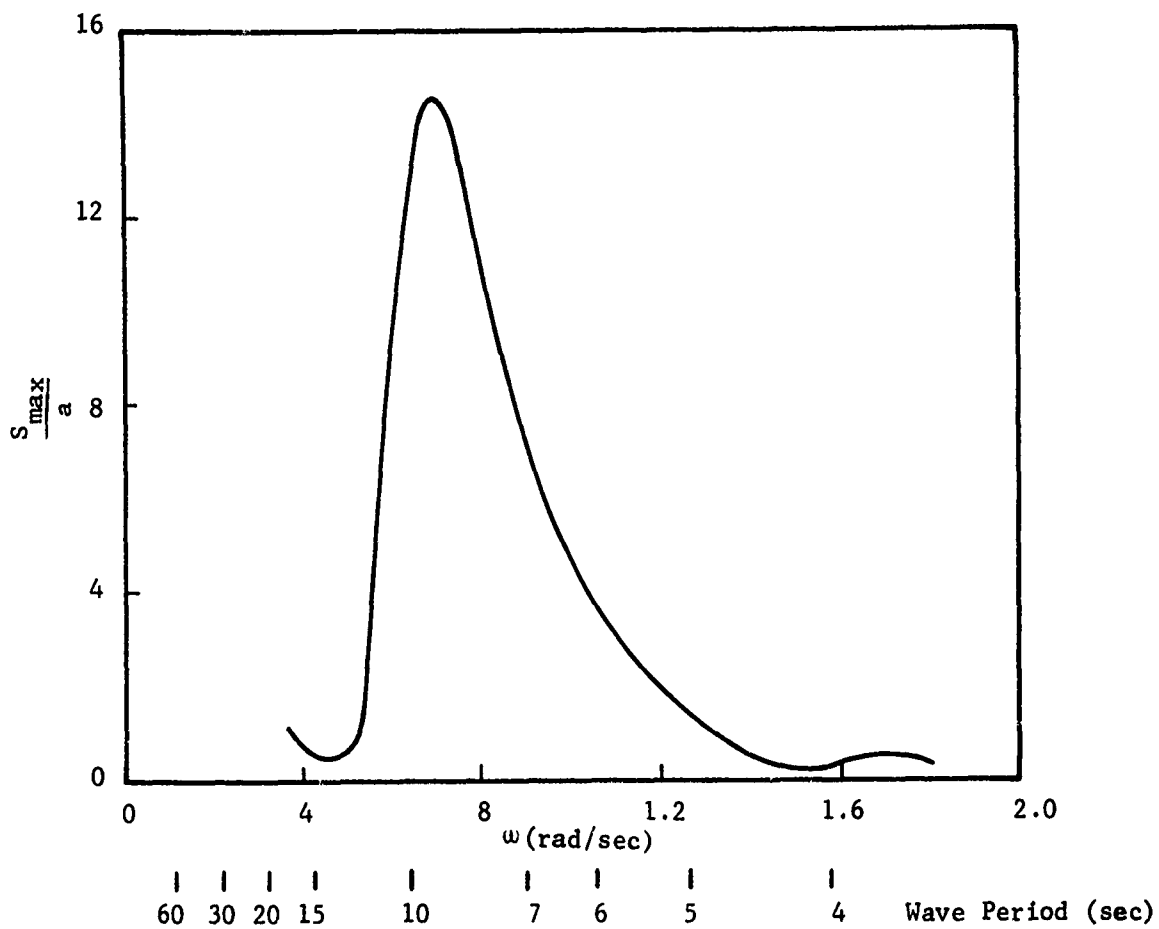[*] The sea is described by a fully developed Pierson-Moskowitz spectrum.

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Figure 10. Ratio of maximum load displacement amplitude
to incident swell amplitude expressed as a
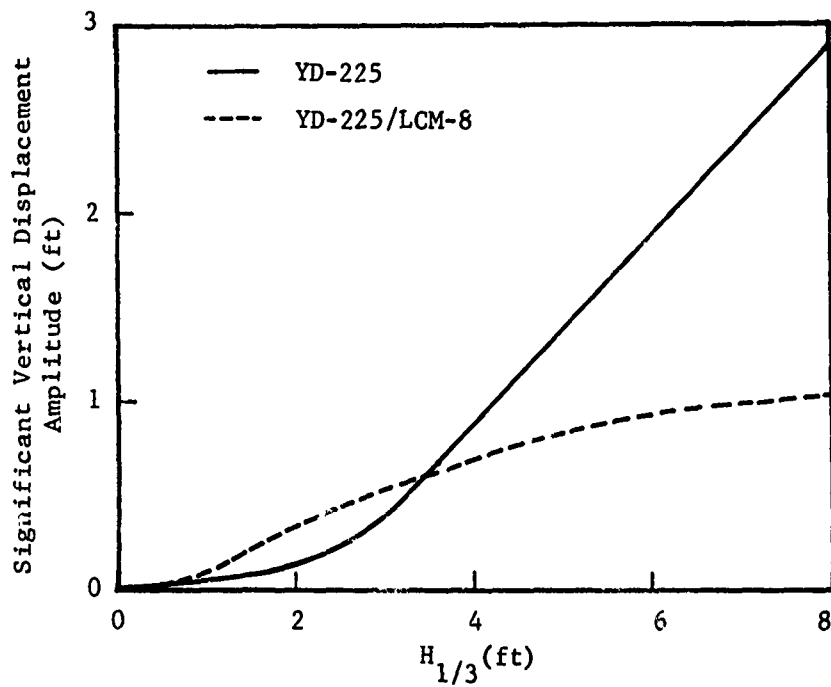function of swell frequency.

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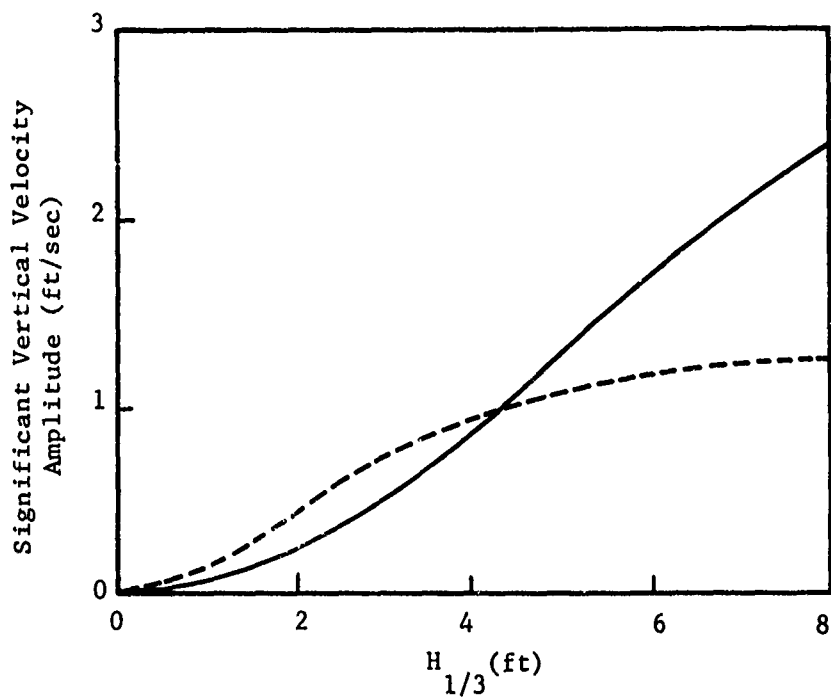Figure 11.   Random crane boom vertical displacement.

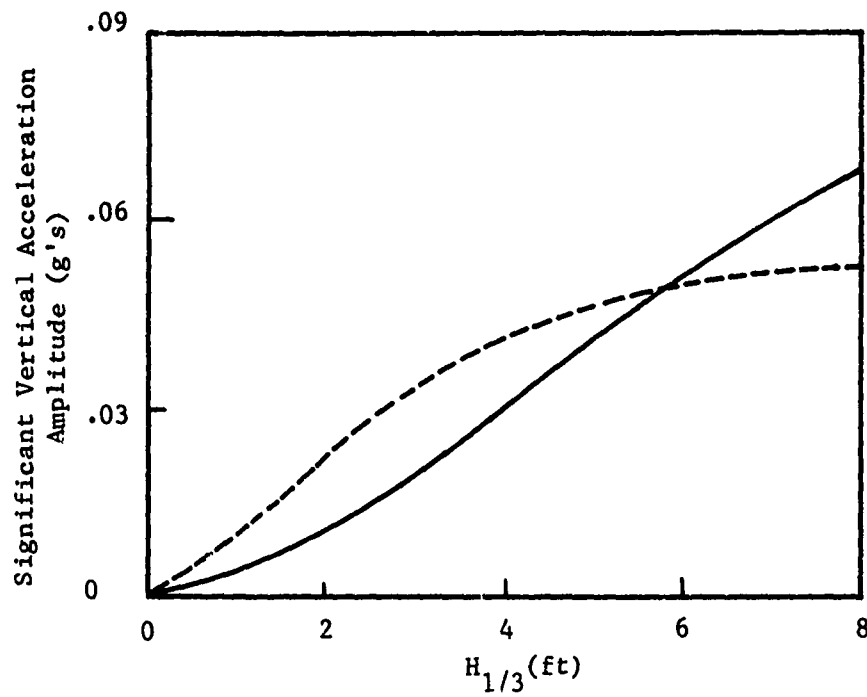Figure 12.   Random crane boom vertical velocity.
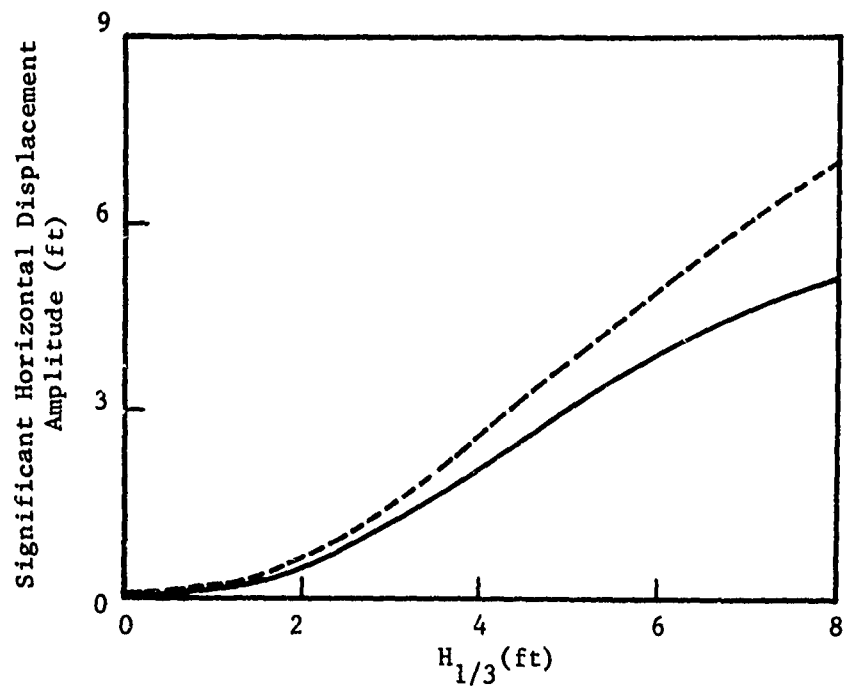
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Figure 13.   Random crane boom vertical acceleration.

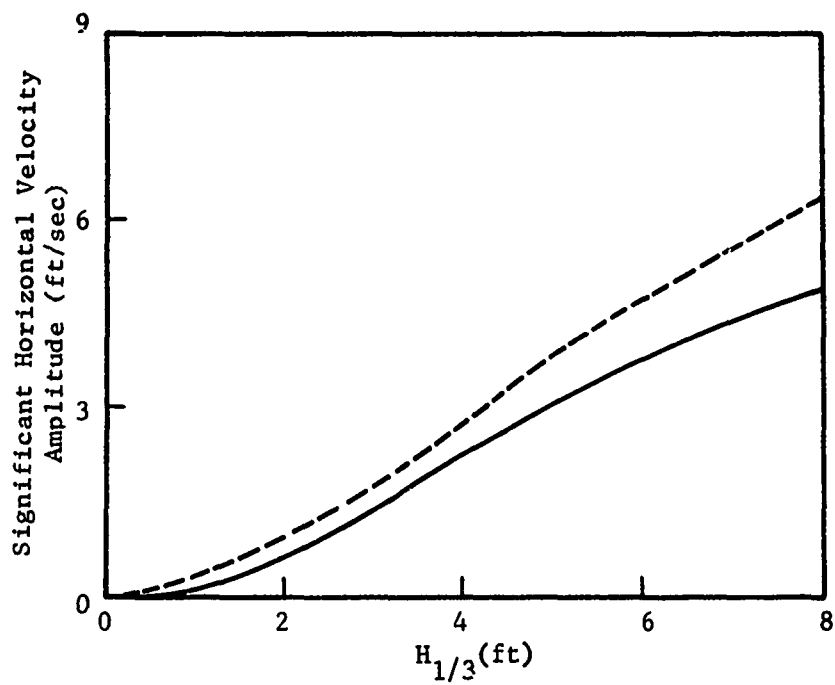Figure 14.   Random crane boom horiztonal displacement.
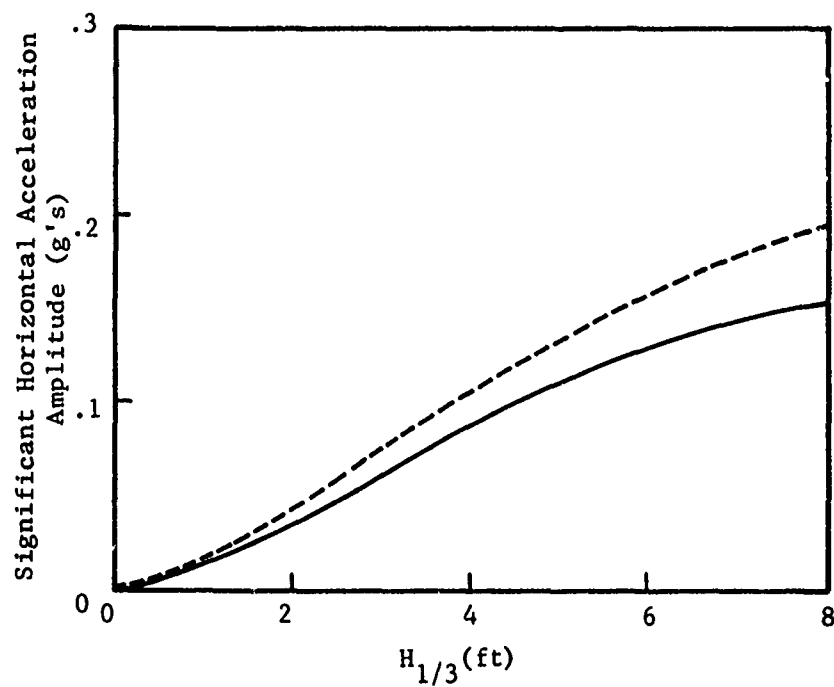
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Figure 15. Random crane boom horizontal velocity.

Figure 16. Random crane boom horizontal acceleration.

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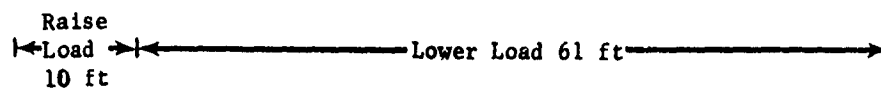Figure 17. Typical simulation of random boom tip and load
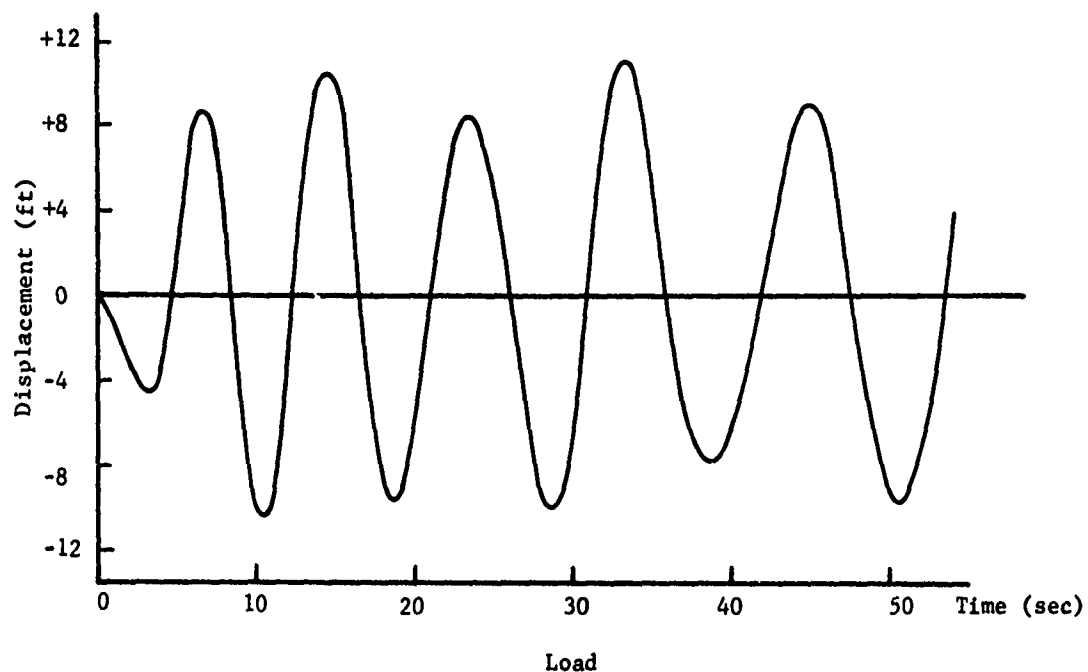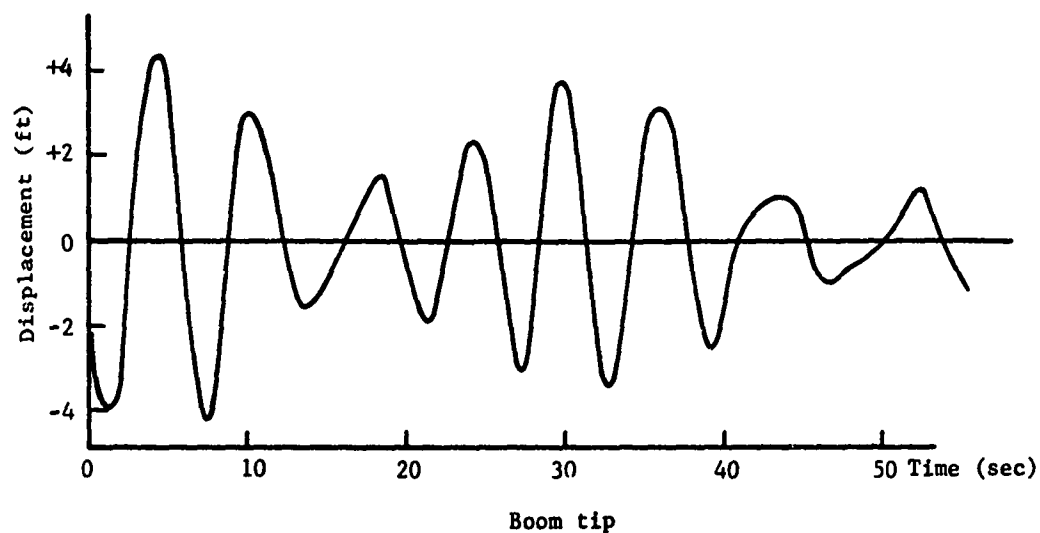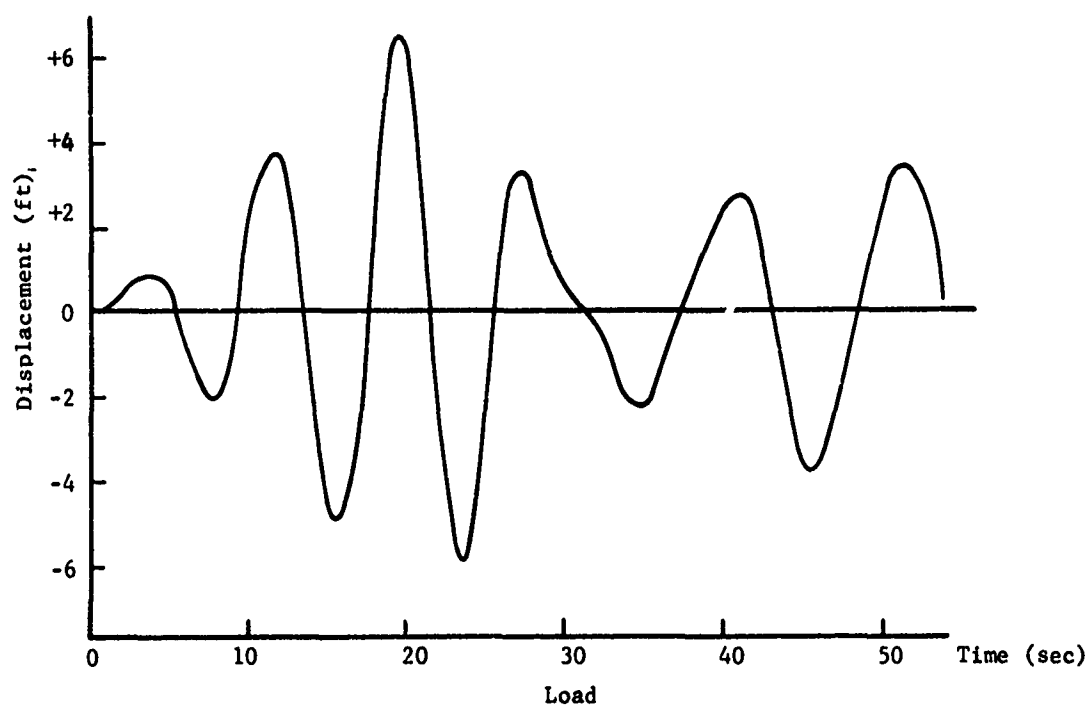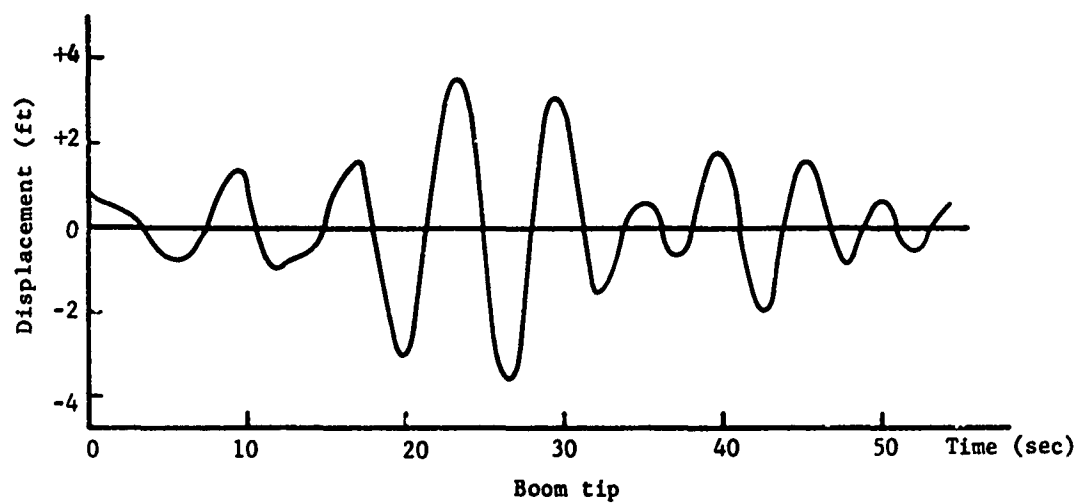displacement ($H_{1/3}$ = 5.0 ft)

Figure 18. Typical simulation of random boom tip and load
displacement ($H_{1/3}$ = 5.0 ft)
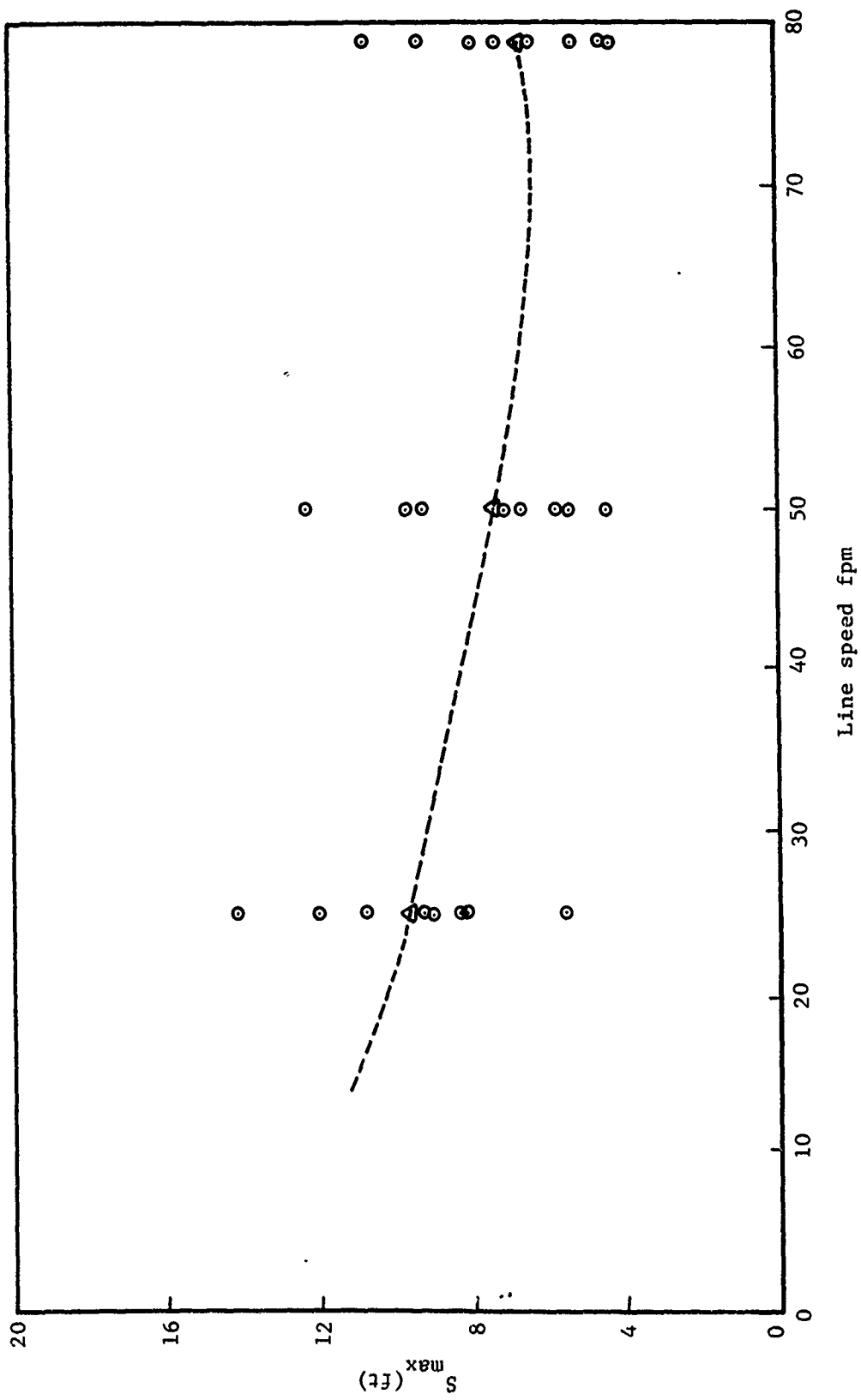
Figure 19. Maximum load displacement amplitude in random seas as a function of line handling rate ($H_{1/3}$ = 5.0 ft)
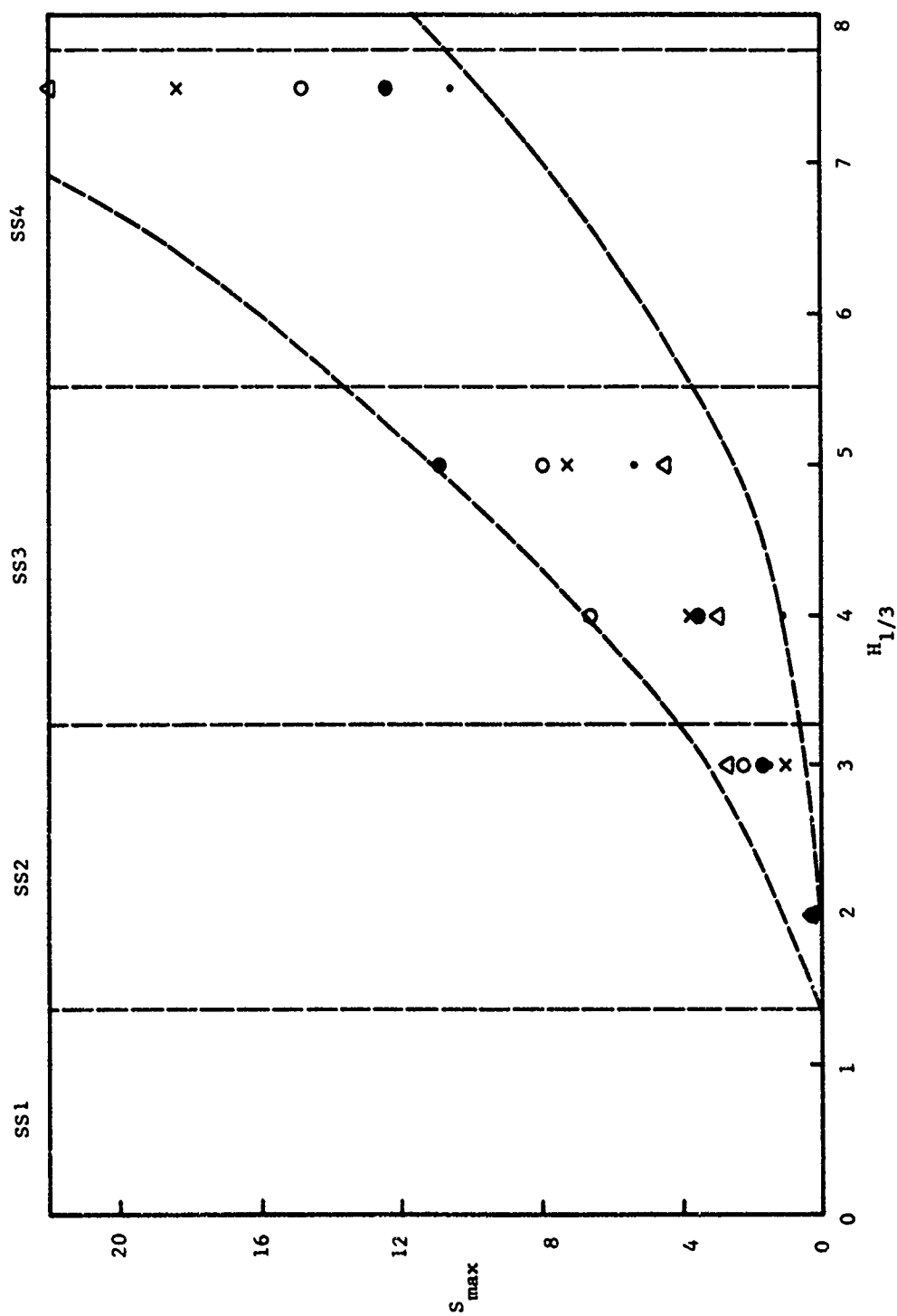
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Figure 20. Maximum load displacement amplitude in random seas as a function of sea state (line rate = 79 fpm).

Appendix A is a brief summary of field tests which were conducted using a Navy 100-ton yard crane to unload containers from an LST. The results from these tests are highly pertinent to topics discussed in this report. Appendix B consists of several example problems which are solved using the graphical results that have just been presented.

FINDINGS AND CONCLUSIONS

1. A theory has been developed for predicting the horizontal response of an unrestrained wire suspended load in regular and random seas. The line length is allowed to vary; thus the load response in random seas is a non-stationary random process.

2. The non-stationary nature of the load response in random seas, coupled with the short duration of each lift cycle (1-2 minutes), requires multiple load response simulations for accurate results. Additional study is required to relate accuracy of prediction with total simulation time.

3. Results from the analysis for a Navy 100-ton yard crane, operating at a maximum line rate of 79 fpm, indicate that the maximum load displacement amplitude to be expected in a sea state 3 is at least 2 times as great as the significant wave height ($H_{1/3}$ = 5.0 ft).

4. The corresponding maximum load displacement amplitude to be expected in 5-foot high, 9-second period regular swell (critical swell period for the crane) is about 7.5 times as great as the swell height.

5. The predicted unrestrained load response for the 100-ton crane is clearly unacceptable for reasons of safety and for the adverse effect it would have on the rate of cargo transfer. Positive tagline control is required at all times to control load pendulation.

6. Faster line handling rates have a mitigating influence on unrestrained load oscillations. For the 100-ton crane, the maximum load displacement amplitude for a 79-fpm line rate is about half of that for a rate of 25 fpm.

FUTURE PLANS

1. Extend the theory to include motion in more than one plane.

2. Investigate predictive accuracy in random seas as a function of total simulation time.

3. Extend the theory to include the effects of linear tagline restraint.

Appendix A

OSDOC TESTS AT CORONADO

In March 1972, tests of an at-sea container transfer system using present fleet components were conducted at Coronado, California. These tests were in support of the joint Navy/Army Offshore Discharge of Containers (OSDOC) program. The tested system consisted of a Navy 100-ton yard crane (YD-193 - same class as the YD-225), an LST which functioned as a mock containership, a 6x15 receiving barge constructed of T-series steel pontoons and a four section pontoon causeway ferry.

The crane was used to transfer containers from the LST to the 6x15 receiving barge. A heavy duty forklift then lifted and positioned each container onto a flatbed truck aboard the causeway ferry. After four containers had been loaded onto trucks, the causeway ferry proceeded to the beach discharge area.

Sea conditions encountered during the two days of testing at sea are summarized in Table A-1. The predominant swell was quite regular and unidirectional. The natural period of the suspended containers during the transfer operations was around 10 seconds. Since this period is close to the period of wave excitation, it can be seen that load pendulation was <u>potentially</u> a serious problem. With one exception, however, pendulation was effectively controlled through the use of hand held taglines, one line at each corner of the container spreader bar. By wrapping the lines around deck cleats, considerable pendulation restraining force could be developed by each line handler. On one occasion, shortly after the container was lifted from the deck of the LST, positive tagline control was lost, and the container did exhibit an appreciable amplitude of oscillation (8 - 10 feet). Quick action by the crane operator and line handlers brought this container under control, and the transfer was completed without further incident.

The tests at Coronado demonstrated the importance of having constant tagline restraint to prevent pendulation. A large expanse of deck area aboard the LST, crane and 6x15 barge provided tagline handlers with considerable freedom of movement. Cleats and other tie points were abundant and accessible. The crew handling the taglines and the crane operator were experienced and skilled. In future cargo transfer operations in the open sea under less ideal conditions, one can speculate that cargo pendulation will prove to be a more troublesome problem than it was at Coronado. In fact no other problem is likely to have a more adverse effect on the cargo unloading rate and to present a greater hazard to personnel and equipment.

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Table A-1. Swell and Sea Conditions During
OSDOC Tests

Date	Time	Maximum Swell Height Observed	Several Swell Heights of --	Many Swell Heights of --	Average Swell Period	Short-Period Waves
22 Mar	1115-1145	3½	---	1½-2 ft	12	Some, low
	1145-1215	4½	---	1½-2 ft	12	Some, low
23 Mar	1015-1030	5	3½-4½ ft	2	8	Some
	1030-1100	5½	---	3½-4½ ft	9	Some

Appendix B

EXAMPLE PROBLEMS

The graphs presented in the body of the report will now be used to solve typical problems which might be encountered in cargo transfer operations using the YD-225 class floating crane.

__Given__: A YD-225 class floating crane is moored stern-on into the incident sea. The water depth is 100 feet, and the boom is in the same position as that aboard the vessel cited earlier in the text. All loads will be handled at the maximum line speed on 79 fpm.

__Find__: The horizontal boom tip displacement and the maximum unrestrained load amplitude in regular swell having a height of 6 feet and a period of 9.5 seconds.

__Solution__: From Figure 5, one reads a horizontal boom tip operator of 0.90. The product of this operator and the incident swell amplitude yields the estimate for the horizontal boom tip displacement amplitude, viz;

$$0.90 \times 3.00 = 2.70 \text{ feet}$$

The total boom excursion is twice this value, or __5.40 feet__.

The maximum unrestrained load amplitude in 9.5 second swell can be obtained readily from Figure 9. Thus, for a period of 9.5 seconds, one reads a load response amplitude operator of 15.8 ft/ft. When this operator is multiplied by the boom tip displacement amplitude, the estimate of the maximum load displacement amplitude becomes:

$$2.70 \times 15.8 = \underline{42.7 \text{ feet}}!$$

The third trace in Figure 8 indicates that this extreme boom tip excursion occurs around 50 seconds after the beginning of the load handling cycle when the load is suspended 107 feet below the boom tip.

__Find__: The horizontal boom tip displacement and the maximum unrestrained load amplitude in a fully random sea state 3 ($H_{1/3}$ = 5.0 ft).

__Solution__: The significant horizontal displacement amplitude of the boom tip can be read directly from Figure 14 as 3.00 ft. The total significant excursion is twice this estimate, or __6.00 ft__. An estimate

of the average of the 1/10 highest or 1/100 highest boom excursions can be obtained by multiplying the significant value by the following coefficients:[*]

$$B_{1/10} = \underline{1.27}\ B_{1/3}; \qquad B_{1/10} = \underline{7.62\ ft}$$

$$B_{1/100} = \underline{1.67}\ B_{1/3}; \qquad B_{1/100} = \underline{10.02\ ft}$$

Due to the non-stationary condition imposed by raising and lowering the load, it is impossible to determine an RMS or significant swing amplitude. What one must do, then, is to exercise the computer program employing different sets of random wave phase angles, the more sets the better. Figure 19 depicts the results of nine different runs for the YD-225. The maximum and minimum swing amplitudes observed for a 79 fpm line rate are 10.8 ft and 4.4 ft, respectively; the mean of all runs is 6.7 ft.

Find: When the line length is at its greatest extension (116 ft), determine the force required to prevent a load from pendulating: (1) in regular swell having a height of 6 feet and period of 9.5 seconds and (2) in a fully random state 3 sea ($H_{1/3}$ = 5.0 ft). What is the maximum restraining force in 6 foot high swell?

Solution: The required force is simply:

$$F = W\ \tan \alpha$$

where W is the weight of the load and α is the angle that the lifting line forms with the vertical. Since $\tan \alpha = B_a/L$, where B_a is the horizontal boom tip displacement amplitude (2.70 feet) and L is the line length, the required force is

$$F = \frac{2.70W}{115} = .024W$$

Thus, a 20-ton container would require a horizontal restraining force of about 960 lbs.

It is clear from Figure 5 that the maximum restraining force in six foot swell (having a period of 20 seconds or less) will occur at a period of 6.5 seconds. The horizontal displacement amplitude operator for waves of this period is 1.73; and it follows that:

$$F = \frac{(1.67(3.0)}{115.0} \quad W = .044W = \underline{1,760\ lb}$$

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[*] These coefficients can be used with all significant data presented in the report.

Find: A container and its contents has passed a 6-inch drop test. Determine the critical height of 7 second swell which will result in potentially damaging vertical motion between the container and a fixed receiving platform.

Solution: A mass dropped from a height of 6 inches has a velocity upon impact of 5.68 ft/sec. From the solid curve in Figure 3, the vertical velocity amplitude operator for 7.0 second swell is 0.77 ft/sec/ft. Then, swell with an amplitude of

$$\frac{5.68}{0.77} = \underline{7.38 \text{ feet}}$$

will cause impact stresses exceeding those experienced during the drop test.